

TEMPUS JEP DEREK
Environmental Study

HYDROLOGY AND HYDRAULIC STRUCTURES

Lecture notes prepared by

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Hydrology and Hydraulic Structures

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Hydrology

This text-book is prepared especially for the introductory course in environmental and resources engineering study within the TEMPUS Joint European Project 2004. The length and depth of coverage is satisfactory in one-semester sequence. Briefly, Hydrology and Hydraulic structures text-book is divided into two parts.

The first module - Hydrology provides a review of hydrological principles and their application, followed by sedimentation dynamic in watershed scale, and some specialized topics in groundwater and urban hydrology. The second part deals with design of hydraulic structures for erosion, sedimentation and flood control.

1. Introduction to hydrology

Upon completion of this lesson, the student will know the history and the development of hydrology as a science and its linkages with other natural sciences. He/she will be able to distinguish various components of the hydrological water cycle and to establish water balance. Introduction to hydrological data needs, collection and storage is presented.

Hydrology is a science related to water. Another definition is that hydrology is a science for observation and measurement of water in the atmosphere, on the land and below the land surface. Hydrology also can be defined as a science that underlies the development and control of water resources. Webster's Third New International Dictionary (*Merriam-Webster*, 1961) describes hydrology as "a science dealing with the properties, distribution, and circulation of water, specifically, the study of water on the surface of the land, in the soil and underlying rocks, and in the atmosphere, particularly with respect to evaporation and precipitation". It is an interdisciplinary science which borrows and integrates many other science branches, such as: physics, chemistry, biology, geology, fluid mechanics, mathematics, and statistics.

Hydrology is divided into the following branches: hydrometeorology (water in the atmosphere), potamology (surface water/streams), limnology (lakes), criology (snow and ice), glaciology (glaciers), oceanology (oceans), and geohydrology (water in the lithosphere/groundwater). The hydrological investigations of the atmosphere are very close related to meteorology and climatology. *Alan L. Prasuhn* (Fundamental of Hydraulic Engineering, 1992) divides hydrology into physical hydrology and statistical hydrology. Physical hydrology describes the physical processes by physical laws and physical observation. Statistical hydrology is based on probabilistic laws. Both of these topics are necessary tools in understanding the subject that is included in graduate and undergraduate civil engineering courses.

For practical reasons hydrology is sometimes limited in various respects, and do not cover all studies, for example oceans are main subject in oceanography, or medical water is main concern of medical hydrology. Hydrology has many practical applications and to emphasize its importance the term "Applied hydrology" is commonly used. Since these applications are in the field of other engineering branches (hydraulic, sanitary, agriculture, water resources) the term "Engineering hydrology" is also used. Hydrological phenomena in urban areas are specific due to construction surfaces and drainage and for this practical application the term "Urban hydrology" is used. These practical applications of hydrology are basis in structural design, water supply, waste water disposal and treatment, irrigation, drainage, hydropower, flood control, erosion and sedimentation control, fish and wildlife preservation, coastal works, and other engineering works.

1. 1 Historical review

Hydrology is a relatively new science. Its historical development can be viewed through a different periods. In Handbook of Applied Hydrology (*Ven Te Chow*, 1964) these periods have been discussed upon the activities and achievements in hydrology. Period of speculation (Ancient-1400) is a period when hydrological cycle was speculated by many philosophers (*Homer, Thales, Plato, Aristotle*). The Renaissance (1400-1600) is a period of observation when pure philosophical concepts changed toward the observational science (*Leonardo da Vinci, Bernard Palissy*). Next century (1600-1700) is known as a period of measurements when modern hydrology begun to develop. Within this period the measurements of rainfall, evaporation, capillarity, flow velocity, and discharges have started (*Pierre Perrault, Edme Mariotte*). In the period of experimentation (1700-1800) hydraulic experimental study of hydrology begun and understanding of hydraulic principles was obtained (*Pittot tube, Woltman's current meter, Borda tube, D'Alamber principle, Bernoulli theorem, Chezy formula*). Period of modernization (1800-

1900) is a period when the great contributions in surface water measurements and groundwater hydrology were achieved (*Kutter, Manning, Dalton, Miller*). Hydrometry was greatly advanced through development of instruments and methods of measurements, and the systematic surface water measurements has begun. During this period several governmental hydrological agencies were founded (U.S. Army Corps of Engineers, Weather Bureau). In the period of empiricism (1900-1930) hundreds of empirical formulas were proposed since the physical basis for most quantitative hydrological determinations were not well known, and during this period many international organizations were founded, such as International Association of Scientific Hydrology (IASH), and International Union of Geodesy and Geophysics (IUGG). Period of rationalization (1930-1950) is a period when rational analysis instead of empiricism in solving hydrological problems was developed (*Sherman* demonstrated the use of unit hydrograph, *Horton* demonstrated a new approach in determination of rainfall excess on the basis of infiltration theory, *Gumbel* proposed the use of extreme-value distribution for frequency analysis of hydrological data). The last period is the period of theorization (1950-date) when theoretical approaches and computational modeling in hydrological problems have been used extensively. During this period many international activities in hydrology and water resources were developed, such as United Nations Educational, Scientific and Cultural Organization (UNESCO), World Meteorological Organization (WMO), Food and Agricultural Organization (FAO), World Health Organization (WHO), and the International Atomic Energy Agency (IAEA).

1.2 Hydrological cycle

The hydrological cycle represents water circulation and transformation in the three parts of the earth system: atmosphere, hydrosphere, and lithosphere. The atmosphere is a gaseous envelope above the earth surface. The hydrosphere is the water body on earth surface, and the lithosphere is a solid rock under the hydrosphere. Since the water is renewable substance/material the hydrological cycle has no beginning or end. The water evaporates from the land and the oceans and as a water vapor becomes a part of the atmosphere. The water vapor rises due to orographic lifting, frontal lifting, and convective air heating. It cools, condenses, and finally precipitates to the earth (land and oceans). The precipitated water (rain, snow) may be intercepted or transpired by plants, may run over the ground surface and in the streams, or may infiltrate into the ground. Thus, the hydrological cycle may be described as a simple link between different arcs, but represents also very complicated processes of evaporation, precipitation, infiltration, transpiration, percolation, and runoff, Figure 1.1. The descriptive diagram of hydrological cycle represents two small cycles, land-atmosphere-land and ocean-atmosphere-ocean, and one big cycle ocean-atmosphere-land-ocean.

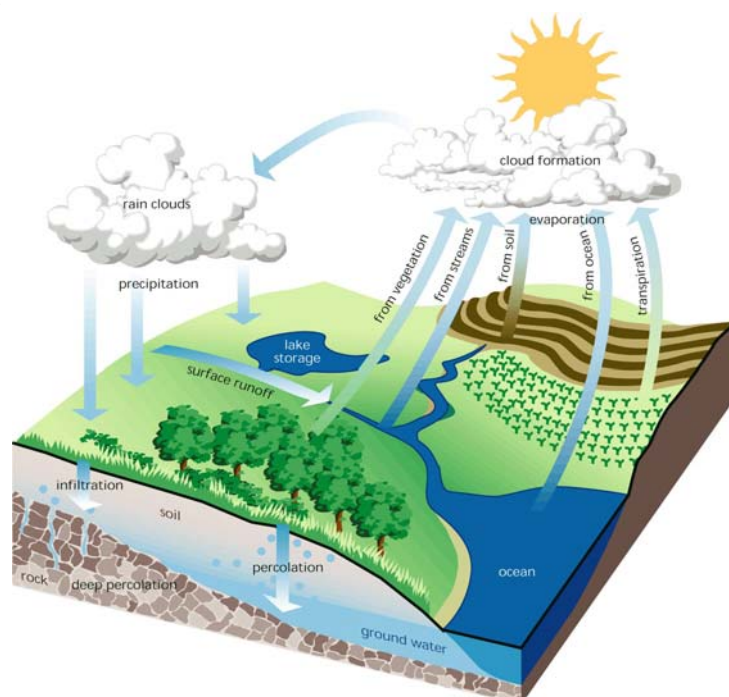


Figure 1.1 Hydrological cycles

The water quantities in hydrological cycle may be evaluated by the simple water balance equation, which is known also as hydrological equation:

$$I-O=\Delta S \quad (1.1)$$

where I is the inflow over a given period (total precipitation over the area, total inflow runoff, total groundwater inflow), O is outflow (evaporation, transpiration, total surface outflow, groundwater outflow, water use), and ΔS is the change in storage. This equation essentially is a form of continuity equation. Each component in this equation may be expressed in (m^3/s), (mm/s), (m^3), or in (mm). The inflow and outflow may be expressed with the following:

$$I=(P+K)+(D_1+D_2+D_3)+R \quad (1.2)$$

$$O=(E_1+E_2)+(S_1+S_2+S_3)+(N_1+N_2) \quad (1.3)$$

where P is precipitation, K is condensed water, D_1 is surface water inflow, D_2 is groundwater inflow through porous soil masses, D_3 is groundwater inflow from karstic masses, R is artificially conveyed water from other area, E_1 is evaporation from water surface, E_2 is evaporation from ground, S_1 is surface water outflow, S_2 is groundwater outflow through porous soil masses, S_3 is groundwater outflow from karstic masses, N_1 is artificially drained water from the area, and N_2 is irretrievable used water. Obtaining all these components is extremely difficult and very often the water budget of the region is analysed by neglecting some of the above mentioned components.

The water distribution in a different Earth spheres are presented in Table 1. According to these quantities it is obvious that 97% of all water in the world is contained in the oceans (hydrosphere). Only 0.001% of water is contained in the atmosphere, 1.9% in the continents, and 0.5% is contained in the lithosphere as groundwater.

	Quantity [$10^3 \cdot \text{km}^3$]	Distribution [%]
Atmosphere/Atmospheric vapor	13,00	0,001
Hydrosphere/Oceans	135.400,00	97,583
Continent/Land:	26.431,70	1,910
- rivers	1,70	
- lakes	125,00	
- sees	105,00	
- soil moisture	150,00	
- biomass	50,00	
- ice and glaciers	26.000,00	
Lithosphere/Groundwater	7000,00	0,506
Total	1.383.844,70	100,00

Table 1.1 Water distribution in different Earth spheres

1.3 Hydrological data

In order to make hydrological computations and analysis, to establish design criteria and to make forecast the hydrologists need data. Data have to be collected, processed and stored in such a way that they can be used easily. The sources for obtaining hydrological data are:

- National and regional archives or libraries of agencies responsible for data acquisition and publication, usually these are authorized hydrometeorological services that have to be connected with world meteorological observation system (UNESCO/WMO, 1977)
- Private organizations such as power authorities or companies having interest in hydrological measurements

- Research papers and project reports
- Survey reports of research and development agencies
- Field observations
- Maps on related topics

Hydrological data are usually time series. A distinction is made between time series of constant time step and data with varying time step. Data on water levels, flows and rainfall are normally time series at equal time step.

To collect data a *hydrological information system* is needed which is consisting of observation network, hydrological data base and hydrological models.

- The *observation network* should allow the observation of all relevant components and processes of the hydrological cycle at a sufficiently accurate density, both in time and in space.
- The *hydrological data base* is used to store, screen, process and retrieve data at relevant time intervals.
- *Models* are used to describe the processes involved in the hydrological cycle and the links between the different components of the hydrological cycle (e.g. rainfall-runoff models, flood routing models, etc).

To be able to make a good use of data they have to be stored in such a way that all possible errors are removed and that the data are accessible. Several steps are required to get a good storage of data: data screening, data correction and completion, data aggregation, data storage, and publication.

Data screening is a necessary step in order to detect and remove data errors. Two types of errors may be distinguished: systematic and accidental.

Observations and measurements in hydrology generate a large amount of data. Data aggregation is condensation of data with shorter time steps to data with larger time steps; for example aggregation of hourly data to daily, monthly or annual data (e.g. rainfall or discharge data). Two types of aggregation are distinguished: Through accumulation (rainfall, runoff) and through averaging (temperature, wind speed, water levels, etc.)

Data can be stored in paper files, computer files and databases. When data are collected over a long period it is advisable to publish them after screening and preprocessing, preferably in yearbooks. A number of hydrological database systems are on the market for storage of hydrological data and time series analysis (HYMOS, HYDATA, LOTUS 123).

Problem 1-1:

Using the water balance equation estimate the groundwater recharge through the karst terrain of the Prespa Lake drainage basin if the drop water level in the lake is 5 cm which corresponds to lost volume of water of 15,7 million m³. The following data are known: drainage basin area $A=1360 \text{ km}^2$; lake water surface area $A_L=313,6 \text{ km}^2$; evaporation from the lake surface $E_L=836 \text{ mm}$; evaporation from ground $E_G=115 \text{ mm}$; precipitation over the lake $P_L=680 \text{ mm}$; precipitation over the ground $P_G=760 \text{ mm}$; and the water surface inflow in the lake $D=345 \text{ mm}$.

2. Meteorology

Upon completion of this lesson the student will be familiar with the meteorological parameters which influence the atmospheric motion and the factors influencing the interaction between the atmospheric and terrestrial water. Methods for estimation and calculation of sun radiation, temperature, air humidity and pressure, wind, precipitation and evaporation are presented and discussed.

Meteorology is a science related to the chemical, physical and dynamic processes of the gaseous envelope of the Earth. In its shortest sense it is a science that relates to the atmosphere. The word *meteorology* is derived from the words *meteoros* (lofty) and *logos* (science). This section will give an introductory outline of the basic meteorological parameters and their relation to hydrology. These parameters are measured at meteorological station that, Figure 1.2.

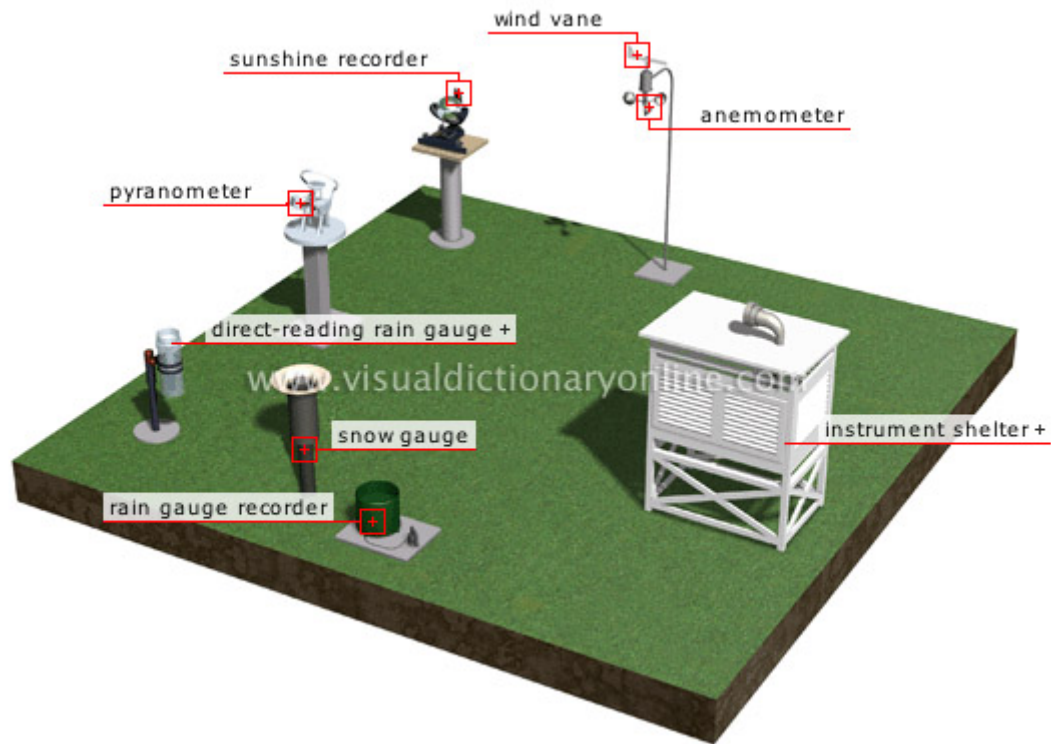


Figure 1.2 Meteorological station

2.1 Radiation

The sun is the source of solar radiation in the atmosphere and on the earth surface which is converting into heat energy. This energy sets in motion all physical properties that impact the hydrological processes. The most impacted process is evaporation from water surfaces (oceans, lakes, rivers). The portion of the total incoming radiation that is reflecting back to space is called *albedo*. Under average conditions the albedo is 40%. The incoming radiation annual amount varies with latitude, and it decreases rapidly with latitude increase. Seasonal variation of the incoming radiation is also observed, in winter is smaller than in summer. The solar radiation intensity is measuring with pyranometers (antimometers) and its duration with sunshine recorders (heliographs).

2.2 Temperature

The incoming radiation that is absorbed by the earth is used to warm the air and the surface substance, and the part of the heat is conducted away from the interface. The conductive capacity of air varies from 0.01 to 1.0, and for water it varies from 0.3 to 17, depending upon stability. For other surface substances it varies from 0.002 (snow) to 0.04 (soil). Since the heat capacity of water is very much larger than that of the air, the oceans are effective heat reservoirs, and the temperature range over land is much greater than at the ocean.

The air temperature depends of the latitude, orographic characteristics, precipitation, and other local factors of the region. The difference between maximal and minimal daily temperature is greater in warm period of the year. The minimum is usually in the morning before the sunrise, and the maximum is couple hours after noon. The air temperature decreases with altitude increase, but there some inversion, mainly in winter and foggy days, when the temperature increases with altitude increase. The air temperature is measured discontinuously by thermometers placed at 2.0 m height from the land, and continuously by thermographs.

The land temperature depends of its color, geological composition, moisture, and vegetative cover. During the day wet soils absorb the heat slower than dry soils, while in the night the dry soils cool down faster than wet soils. The land temperature is measured by geothermometers placed at different depths.

The solar radiation is absorbed by the water and the temperature decreases with the depth to 4°C when the density is 1000 kg/m³. The average temperature of the surface water is greater than the average soil temperature. The daily amplitude of water temperature is from 0.5 °C to 3 °C with maximum 3-4 hours after noon, and minimum 2-3 hours after the sunrise. The annual amplitude of the surface water temperature is from 25 °C to 40 °C for rivers and lakes, and between 17 °C and 20 °C for the oceans and seas. Freezing of water occurs when the air temperature is below zero, but the process itself depends on other factors such as flow velocity, depth, and physical properties of the water.

2.3 Humidity

The evaporated water from the oceans, lakes, rivers, soil and vegetation, rises into the atmosphere, makes the air more or less wet, and is called humidity or water vapor. The amount of water vapor in the air may be expressed as the pressure that the vapor would exert if the other gasses are absent. It is usually denoted by the symbol e and is expressed in (mmHg) or (mbar). The maximum amount of vapor is denoted by the symbol e_o and depends upon the temperature. If the air at the observed temperature is containing maximum amount of the water vapor it is said that the air is *saturated*. More precisely definition of the saturation may be explained as state in which water vapor can exist in equilibrium with a plane surface of pure water at the same temperature. The dependence between the maximum water vapor pressure and the air temperature can be defined by *Magnus* with the following equation:

$$\log e_o = \frac{at}{t+b} + 0,786 \quad (2.1)$$

where the constants are $a=7.5$ and $b=277.3$ for water, and $a=9.5$ and $b=265.5$ for ice. This expression is applicable for the temperature range between +40°C and -40°C. Other humidity measures that are used may be defined as follows. The *relative humidity* U is the ratio between the actual vapor pressure e and the saturation vapor pressure e_o and is expressed in percentage.

$$U = 100 \frac{e}{e_o} \quad (2.2)$$

The air is saturated when $U=100\%$, and the air is dry if $U=0\%$. The *vapor density* ρ_v or *absolute humidity* is the mass of water vapor contained in a unit volume of air:

$$\rho_v = \frac{e}{R_v T} \quad (2.3)$$

where R_v is specific gas constant of water vapor, and T is thermodynamic temperature (Kelvin temperature). If R is a specific gas constant of dry air, then $R_v=(8/5)R$. The term *specific humidity* s is defined as the ratio of water vapor containing in kilogram of air:

$$s = \frac{\rho_v}{\rho_a + \rho_v} \quad (2.4)$$

where ρ_a is density of dry air.

2.4 Atmosphere and air pressure

On the whole, the atmosphere may be discerned with more or less five concentric spheres or shells. The lowest is called *troposphere*. It contains 75% of the mass and almost all moisture and dust. All the phenomena which are called *weather* are addressed to this layer. The top of the troposphere is called *tropopause*, and above it is the *stratosphere*. In the stratosphere the temperature is more or less constant and the stratification is very stable. Much of the ozone discussed bellow is containing here. Above the

stratosphere is warm layer called *mesosphere*. The highest temperature in this layer is greater than the temperature of the ground, even on warm days. This phenomenon is due to the absorption of ultraviolet radiation from the sun. Above the mesosphere is *ionosphere* which base is 70-80 km from the earth. The fifth layer is called *exosphere*. It is the outermost shell and here the atmosphere has lost the property of a continuum.

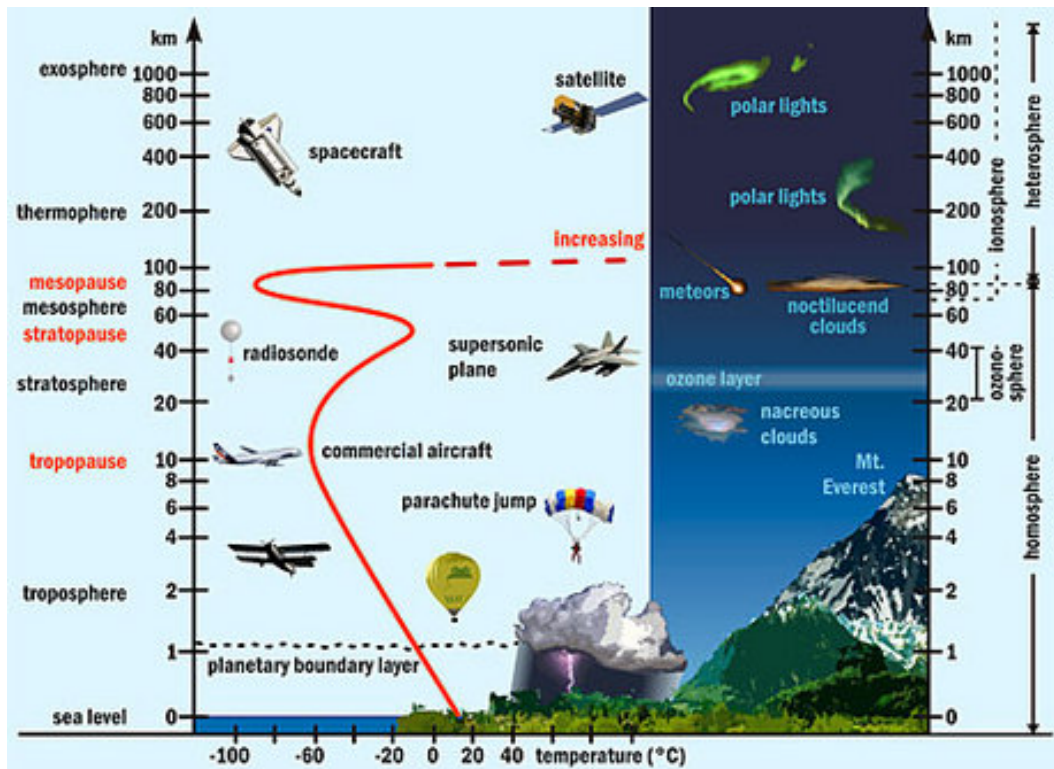


Figure 1.3 Spheres of the atmosphere

The atmosphere is a complex mixture of individual gases. The four principle gases are nitrogen (N_2), oxygen (O_2), argon (A_r), and carbon dioxide (CO_2). Actually, these four gases amount about 99.997% of the whole. The total weight of the atmosphere amounts 5.6×10^{15} metric tones, and the weight of the water vapor is 1.5×10^{14} tones. The diatomic oxygen (O_2) in the atmosphere is about 21%. Triatomic oxygen (O_3), or ozone, is found in a small portion mainly at great heights. The ozone is important because it prevents harmful ultraviolet radiation to reach the levels where biological processes are prominent. The amount of carbon dioxide is not constant. It is produced by the animal world, by burning fuels, by volcanic actions, and various decay processes in the soil. Since the industrialization has begun the amount of carbon dioxide has increased rapidly which causes the rise in the air temperature and the effect of so called “greenhouse”. This phenomenon is not quite understood, but is related to the climate change impact assessment hydrology as a science of observations and measurements plays a great role.

The atmosphere is very near to hydrostatic equilibrium, and the pressure and height may be expressed by the equation:

$$-dp = \rho g \cdot dz \quad (2.5)$$

where g is gravity acceleration $g=9.81 \text{ m/s}^2$, ρ is density of natural air ($\rho = \rho_a + \rho_v$). The atmospheric pressure at sea level under standard conditions amounts 101.325 kPa or 10.3 m of water column. It is measured by barometers and mechanical or liquid manometers. The atmospheric pressure doesn't change in horizontal direction in quiet atmosphere, but it changes in vertical direction. When the altitude is increasing then the pressure decreases due to the decreasing of the air density. At the greater heights this pressure decreasing is smaller. The temperature influences also the atmospheric pressure and it decreases faster in cold air than in warm air. Daily change of the atmospheric pressure is also observed. When the pressure decreases the whether is changing to cloudy and rainy, and if the pressure is increasing the whether is clear and quiet. The atmospheric pressure change has a great impact on evaporation and on

groundwater table oscillation. The corresponding values of pressure and height, height and density, height and temperature are obtained from tables in the literature.

2.5 Wind

The motion of the air in horizontal direction is called *wind*. The motion is result of thermally active substances in the atmosphere. In practice the wind is defined with direction and intensity or speed. The wind direction is noted with **N** (North), **E** (East), **W** (West), and **S** (South). Combining these can be obtained 32 different wind directions, such as **NE** (North-East), **SE** (South-East), **WSW** (West-South-West), and etc.

The diagram of measured wind directions in a region for a given period is called *wind rose*, and may be presented for 32, 16 or 8 world sides. The wind rose with 8 directions is shown in Figure 2.1. The wind characteristics are measured by anemometers (intensity), and by anemographs (direction and intensity) which are usually placed at height 10 m from the ground to avoid the ground surface influence. According to the Bofor scale the wind intensity is classified in 13 grades from 0 ($V=0$ km/hr) to 12 ($V>125$ km/hr), and **C** is used for calm weather.

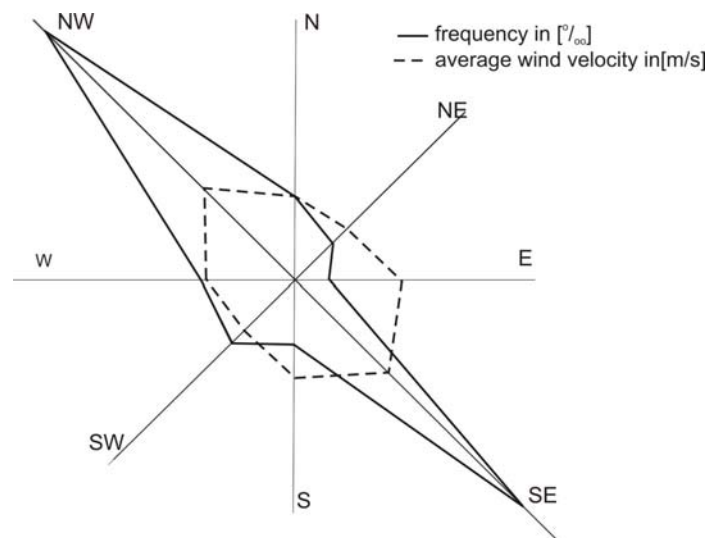


Figure 2.1 Wind rose with 8 directions (Meteorological station SHTIP-Macedonia)

The weather systems and regimes are closely related to air mass motion. Most frequently used terms are front, cyclone, anticyclone, hurricane, tornado, and convection. The border region between adjacent air masses are called *front*. The fronts are classified according to their motion. A cold front is a front along which colder air replaces warmer air, and similarly a warm front is a front along which warmer air replaces colder air. *Cyclone* is a large low-pressure area with the attendant winds. Cyclones are usually accompanied by clouds and precipitation. The *anticyclone* is a large high pressure area with the attendant winds. In general, anticyclones are accompanied by clear/fair weather and moderate winds. The intense cyclone of tropical origin and of relatively small horizontal dimensions is called *hurricane*. A *tornado* is a violent whirl of destructive winds which is accompanied by a funnel cloud. The *convection* is a vertical overturning resulting from static instability. The term convective weather is related to weather that includes showers, squalls, thunderstorms, etc.

The differences in heating of the earth's poles cause the air masses motion known as permanent winds (passate, antipassate, west wind, and north wind). The winds that blow with changeable direction in time are known as periodical winds (mountain winds, valley winds, monsoons). Mountain and valley winds are developed in response to the diurnal heating of mountainsides. On sunny days a gentle breeze is often observed to blow upslope, and on clear nights the wind reverses its direction due to the cooling off the mountainsides. Monsoons are winds with an annual oscillation, blowing from oceans to continents in summer and in reverse direction in winter.

2.6 Precipitation

The mass of water or ice that falls to earth is called *precipitation*. Rain and snow are of greatest importance to hydrology and engineering, especially their amount and distribution. The precipitation is related to the condensation of water vapor in the atmosphere. The rain is a liquid precipitation and can be classified as drizzle, light rain, moderate rain, and heavy rain. Drizzle is a fine sprinkle of very small and rather uniform water drops with diameter less than 0.5 mm and its falling intensity is <0.1 cm/h. Heavy rain intensity is >0.8 cm/h. Freezing rain is called glaze and it appears when rain falls into a cold air and freezes when it strikes the ground. Other forms of precipitation that are significant in meteorology include snow, hail, sleet, mist and fog. Snow is precipitation of solid water, mainly in the shape of branched hexagonal crystals, or stars. Hail consists of concentric layers of ice that build up to large diameters due to buffeting and continued suspension by turbulence in the moisture and freezing atmosphere. Sleet is melting snow or a mixture of snow and rain. Mist and fog are airborne droplets that remain suspended in the air because they are so small.

Since the precipitation is caused by water vapor condensation in the clouds, one should know more about clouds. The basic groups of clouds are cirrus, cumulus, and stratus. Some the other types of clouds are altocumulus, altostratus, nimbus, cumulonimbus, and nimbostratus. *Cirrus* clouds occur in the upper troposphere at approximate height 8-10 km, Figure 2.2. They consist of fine ice crystals and have a delicate silky appearance without shadows. *Cumulus* is a thick cloud whose upper surface is dome-shaped and whose base is more or less horizontal. Altocumulus is cloud with lower sheet, larger flakes, and with light shadows. *Stratus* is a uniform layer of low fog like cloud which does not touch the ground. Altostratus is a dense sheet of gray color often with fibrous structure. Clouds of this type are found in the middle troposphere, and they are usually followed by continuous precipitation. *Nimbus* clouds are found at height up to 4 km and the rain as the most frequent precipitation is falling from these clouds, Figure 2.3. Nimbostratus is a dense, shapeless, and ragged layer of clouds. It is often connected with altostratus at higher levels. *Cumulonimbus* is referred to the great masses of clouds rising like mountains.



Figure 2.2 Cirrus clouds



Figure 2.3 Nimbus clouds

Out of the total precipitation that falls on the earth, about 35% comes from land area evaporation, while the remaining 65% comes from ocean evaporation. Roughly 25% of this water returns in the oceans as runoff while 75% returns directly to the atmosphere by evaporation and transpiration. Precipitation is measured by nonrecording and recording gauges. The most common types are designed primarily for rainfall, but the recording gauges may be used for snowfall measurement as well. By the nature, nonrecording gauges give only the total rainfall between readings, and if more information is necessary, recording gauges are required. An oil film is added to the rain gauges to reduce evaporation and antifreeze is used in winter to prevent freezing. Accurate rainfall measurement is not easy because they are influenced by many factors, such as latitude, sea nearness, relief, forestation, settlements, and industrial capacities. The annual rainfalls usually follow the basic laws in physics. The rainfall amount increases with altitude increase. For Europe this relation is observed to altitude 3-3.5 km, and after it stagnates.

In remote areas or in areas where increased spatial or time resolution is required radar and satellite measurement of rainfall can be used. Radar works on the basis of reflection of the energy pulse transmitted by the radar which can be elaborated into maps that give the location (plan position indicator-PPI) and the height (range height indicator-RHI) of the storms.

2.6.1 Average precipitation over a region

The records of rainfall gauges are expected to represent a large area. Low accuracy is expected if one gauge record data is used. The amount of precipitation can be estimated more accurately if more than one gauge is in or near the area of interest. The average precipitation over a region may be estimated by different methods: arithmetic average method, *Thiessen* polygon method, and isohyetal method.

The arithmetic average method is expressed with the following equation:

$$P_{av} = \frac{\sum_{i=1}^n P_i}{n} \quad (2.6)$$

where $\sum P_i$ is sum of precipitation of all rain gauge stations in the region, and n is the number of stations. A straight arithmetic average of the gauge results is usually inadequate due to the nonuniform distribution of the rain gauge stations.

A better method is *Thiessen* polygonal method which utilizes a weighted average based on the assumption that the precipitation in an area nearest a particular gauge is best represented by the gauge. The procedure, which is illustrated in Figure 2.2, requires that the various precipitation gauge stations first be connected by straight lines. The respective areas, which are to be used as weighting factors, are then defined by constructing perpendicular bisectors of the lines. The polygonal areas that result contain the area closest to each gauge. In this method the more predominate gauge has more weight, while the procedure determines and excludes those gauges that have no influence on the region. Gauges that are outside of the region may also be included if appropriate. The average precipitation by this method is obtained:

$$P_{av} = \frac{\sum_{i=1}^n A_i P_i}{A} = \sum_{i=1}^n a_i P_i \quad (2.7)$$

where A_i is the respective area of the gauge station, P_i is the sum precipitation at the gauge station, A is the total area, and a_i is the ratio A_i/A .

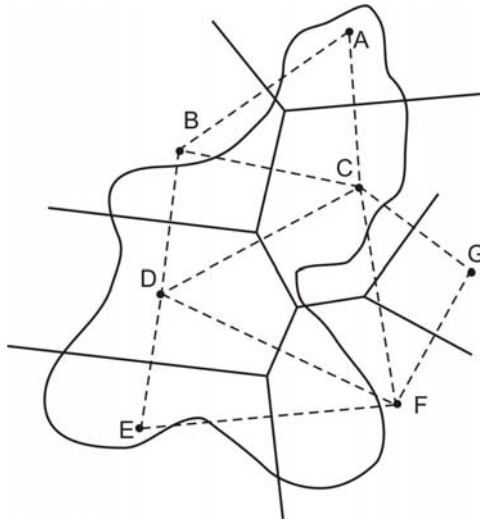


Figure 2.2 Thiessen polygon method

Another averaging method that is commonly used is the isohyetal method, which is based on drawing the lines of equal rainfall amount, or isohyets, presented in Figure 2.3. The average depth is then determined by computing the incremental volume between each pair of isohyets, adding these incremental amounts, and dividing by the total area:

$$P_{av} = \frac{\sum_{i=1}^n \Delta A_i P_i}{A} \quad (2.8)$$

where ΔA_i is the area between two adjacent isohyets. So, the average precipitation is estimated on the relative size of the areas between the isohyets.

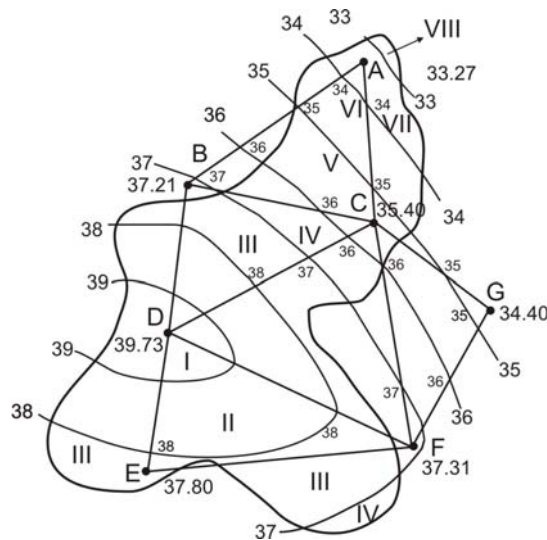


Figure 2.3 Isohyetal method

2.6.2 Probable maximum precipitation

In view of uncertainty involved in frequency analysis and its requirement for long series of observations which are often not available, hydrologists have looked for other methods to arrive at extreme values for precipitation. One of the most commonly used methods is the method of **Maximum Probable Precipitation (MPP)**. The MPP method is based on a physical upper limit to the amount of precipitation that can fall on a given area in a given time. The PMP technique involves the estimation of the maximum limit on humidity concentration in the air that flows into the space above a basin, the maximum limit to the rate at which wind may carry the humid air into the basin and the maximum limit on the fraction of

the inflowing water vapor that can be precipitated. PMP estimates in areas of limited orographic control are normally prepared by the maximization and transposition of real, observed storms. In the areas in which there are strong orographic controls on the amount and distribution of precipitation, storm models have been used for maximization for long-duration storms over large areas. The maximization-transposition technique requires a large amount of data, particularly volumetric rainfall data. In the absence of suitable data it may be necessary to transpose storms over very large distances despite the considerable uncertainties involved. In this case the world envelope curves may be used.

2.7 Evaporation

Water in depression storage, water that infiltrates but remains near the surface, and water in rivers, lakes or any stream is subject to *evaporation*. The process when plants return water to the atmosphere, or the evaporation from plants, is called *transpiration*. Information on this process is essential in agriculture and irrigation. Because of the difficulty in estimation of transpiration and various types of evaporation, they are sometimes combined into a single term *evapotranspiration*. The emphasis in this introduction to the subject of hydrology is limited to the evaporation from water bodies. Evaporation pans are generally used for direct measurement of evaporation. The U.S. National Weather Service has settled on a standard, above ground pan known as the Class A pan.

The evaporation may be calculated by different equations that should be followed by calibration and verification with measured data. *Dalton's law* (1802) proposed an equation where the evaporation is caused by the difference in water vapor pressure:

$$E=C(e_o-e) \quad (2.9)$$

where E is the rate of evaporation, C is mixing coefficient, e_o is maximum vapor pressure, or saturation of vapor pressure at the temperature of water surface, and e is the actual vapor pressure of the overlying air. The greatest difficulty is determination of the coefficient C , which is commonly, but probably inadequately, expressed as a function of wind velocity. The *Fitzgerald* equation (1886) is written:

$$E=(0,40+0,199 w) (e_o-e) \quad (2.10)$$

where w is wind velocity in (m/hr) near the surface. The *Meyer* equation (1915) is written:

$$E=(1+0,1U) (e_o-e) \quad (2.11)$$

where U is relative humidity. Very often in practice is used the *Penman* formula (1948):

$$E=0,35(1+0,24w) (e_o-e) \quad (2.12)$$

The *Thorntwite-Holzman* formula is written:

$$E = \frac{343K_o^2 (e_1 - e_2) (w_2 - w_1)}{(T + 3974,6) \ln \left(\frac{h_2}{h_1} \right)^2} \quad (2.13)$$

where K_o is Karman constant ($K_o \cong 0,4$), e_1 and e_2 are vapor pressures at heights h_1 and h_2 , T is air temperature in ($^{\circ}\text{C}$) at the same heights, w_1 and w_2 are wind velocities in (km/hr) measured at the same heights h_1 and h_2 .

Problem 2-1:

For the drainage basin and rain gauge stations in Figure 2.2 determine the average precipitation over the basin using the Thiessen polygon method. The annual precipitation at the rain gauge stations are: $P_A=77$ mm; $P_B=77$ mm; $P_C=73$ mm; $P_D=93$ mm; $P_E=84$ mm; $P_F=81$ mm; and $P_G=79$ mm. Repeat the same example using the isohyetal method.

3. Drainage basin

Upon completion of this lesson the students will be able to distinguish drainage basin and its characteristic significant for further hydraulic analysis.

The term drainage basin or watershed is used for a land surface through which precipitation is imported. Imported precipitation transformed into runoff leave the system through the basin mouth. The geometry of a drainage basin and its stream system is usually described by measurement of linear aspects, areal aspects and relief aspects. The first two categories of measurement are planimetric, the third category is related to the vertical inequalities of the drainage basin forms. The linear aspects of the drainage network are determined by stream order, stream lengths, and length of overland flow. The stream order system introduced by *Horton* (1932) can be explained as follows. On the channel network map where the valleys are clearly defined and flow lines located, the smallest tributaries are designated order 1. Where two first-order streams join, a stream segment of order 2 is formed; where two of order 2 join, a segment of order 3 is formed; and so on. Example of a stream order classification is shown in Figure 3.1.

After the stream order numbers are assigned, the segments of each order are counted to yield the number N_u of segments of given order u . It is clear that the number of stream segments of any given order will be fewer than for the next lower order but more for the next higher order. The ratio of number of segments of a given order N_u to the number of segments of the higher order N_{u+1} is called bifurcation ratio R_b . The bifurcation ratios range between 3.0 and 5.0 for drainage basins where the geologic structures do not distort the drainage pattern. The theoretical minimum possible value is 2.0. In regions with narrow strike valleys might be expected abnormally high bifurcation ratios.



Figure 3.1 Stream order classification

Another drainage basin characteristic is the coefficient of rotundity:

$$m = \frac{\pi L^2}{4A} \quad (3.1)$$

where L is basin length, and A is basin area. When $m=1$ the basin outline is a circle. Other approximations of the outline shape of the basins may be triangle, when the river network of the basin is developed in upper part or rectangle when the river network is developed all along the main river with short tributaries. The drainage density is simply the ratio of total river segment lengths cumulated for all orders within a basin to the basin area:

$$D = \frac{\sum_{u=1}^{u=k} L_u}{A_u} \quad (3.2)$$

where u is river order, k is the highest river order in the basin, A_u basin area of the river order u , and L_u is its length. Very important analysis of drainage basin is hypsometric analysis. In Figure 3.2 is presented *hypsometric curve* which is obtained by taking the drainage basin to be bounded by vertical sides and horizontal base plane passing to the mouth. The shape of hypsometric curve varies in early geological stages of development of the drainage basin, but once steady state is attained tends to vary little.

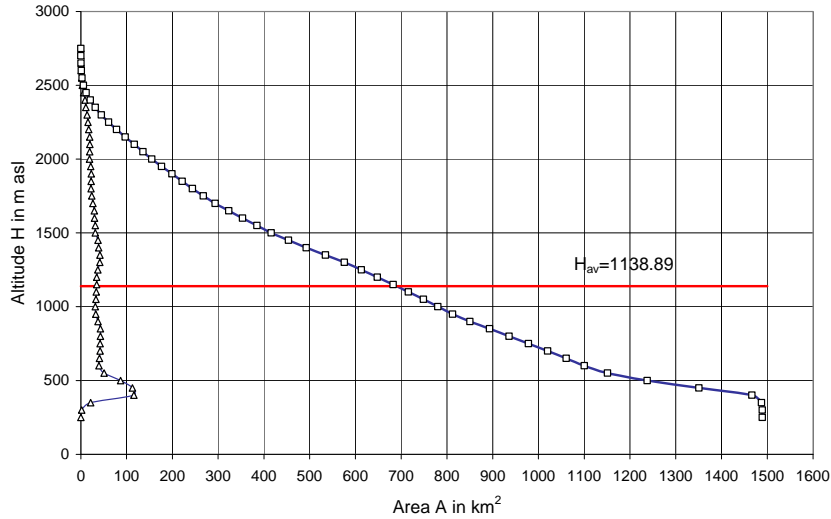


Figure 3.2 Hypsometric curve (Upper Vardar River-section Radusha)

The average height (altitude) of the basin is then obtained:

$$H_{av} = \frac{\sum_{i=1}^n A_i H_i}{A} \quad (3.3)$$

where H_i is the average height (altitude) between two height contours, and A_i is the corresponding area. The average slope of the basin may be obtained by:

$$S_{av} = \frac{\Delta H}{A} \sum_{i=1}^n l_i \quad (3.4)$$

where l_i is the length between two (adjacent) contour lines, ΔH is the height between the adjacent contour lines, and A is the basin area. All these geometrical characteristics of the drainage basins may be analysed by use of Digital Terrain Model (DTM) or any other land development available software. Besides the above mentioned drainage basin characteristics very important in hydrological analysis are the physical-geographical characteristics, such as: geographical location, climate, vegetative cover and land use, geological structure, geomorphologic conditions, and other. The land cover and land use characteristics may be analyzed by the use of GIS based Corine Land Use Classification Model.

4. Measurements in hydrology

Basic hydrological measurements of discharge, water level, flow velocity or sediment content are presented in this chapter. The students will get insight in the way of acquiring usable measured data necessary for further hydrological analysis.

4.1 Discharge

The measurements of basic hydrological parameters are performed on gauging stations that might be recording or non-recording. The continuous measuring is at the recording stations, and discontinuous at non-recording stations. Measuring or estimating the discharge in a river or stream is main objective. There have been developed a number of methods of measuring or estimating the discharge, such as weirs, tracers, electronic meters, direct calculation using the Manning equation, and others.

4.2 Water level

Water level, or elevation of water surface (lakes and other still water bodies) or river stage (rivers, canals) is recording by water level gauge (graduated staff) or by recording gauge (limnigraph). The typical installation in recording gauge station consists of a large stilling well that is connected to the river at a point below the low water. A float within the stilling well is connected by the wire with a drum in the upper chamber. As the water surface elevation changes, the rise or fall of the float rotates the drum. Independently, a pen is driven longitudinally along the drum by a clock mechanism. The result is *stage hydrograph* or continuous record of the water level or river stage. Ultimately, the river stage hydrograph must be converted to a continuous record of discharge called a *discharge hydrograph* or simply a *hydrograph*. This can be done by electronic pressure transducer or air bubbler. The stage may be electronically digitized and transmitted directly to the office receiver and computer. The relation between river stage and discharge is established by a so-called *stage-discharge curve*, characteristic for each cross-section of a watercourse or canal. This is further discussed in Chapter 8.

4.3 Velocity

The discharge itself is determined through the process of stream gauging, which is a graphical integration of the velocity distribution over the cross section. The velocities are measured by *current meter*. In shallow water the meter is positioned in the stream by a wader or supported by cable from above. The current meter operation is based on holding the meter at different height of the selected vertical sections for a time period, usually in excess of 40s, and counting the numbers of revolutions of the wheel. The current meter has its own calibration curve that provides the velocity as a function of the number of revolutions per minute. The common heights where the meter is located are: at surface, $0.2h$, $0.4h$, $0.6h$, $0.8h$, and at the bottom. How many locations will be selected depend of the depth. The velocity distribution in the vertical direction is approximately logarithmic. In practice the average velocity is usually assumed to equal the average of two point velocities obtained at two-tenths and eight-tenths of the depth. When the section is too shallow then a single measurement is obtained at six-tenths of the depth. To gauge a river, the cross section is divided into a number of vertical sections that approximate rectangles. The discharge of each section is calculated by taking the product of the depth, width, and average velocity. The discharge of the river is finally the sum of the incremental discharges.

4.4 Sedimentation

The measurement of sediment transport is a regular but relatively new part in hydrometric measurements. Suspended sediment transport is usually measured by *depth-integrating sediment sampler*. The sampler is designed to cause minimum interference with the flow. Water and suspended sediment enters through the nozzle and are collected in a pint milk bottle. As with current meters, the sampler can be carried by a wader or supported from a bridge by cable. The samples are collected at the same points in vertical sections where the velocities are measured. The samples are then dried and analyzed. Typical

concentration units include weight of sediment per unit weight of sample. The measurement of bed load is usually by pits and various other devices. The bed load sampler or bathometer has been extensively studied and is coming into increased usage. The sample collected from the lower portion of the stream is analyzed similarly to the suspended sediment sample. The unmeasured total load may be estimated by *Meyer-Peter and Muller* equation or by modified *Einstein* equation.

5. Runoff

Upon completion of this lesson, the students will be able to develop relations between the precipitation and runoff by using the factors affecting runoff. In this section the basic relations between rainfall and runoff are discussed.

Runoff is that part of precipitation which appears as flow contribution in surface stream. This is the flow collected from a drainage basin and it appears at the outlet of the basin. According to the source from which the flow is derived, runoff may be recognized as surface runoff, subsurface runoff, and groundwater runoff. The *surface runoff* is that part of the runoff which travels over the ground surface and through channels reaches the basin outlet. The part of the surface runoff that flows over the land surface toward stream channels is called *overland flow*. The *subsurface runoff* or storm seepage is that part of the precipitation which infiltrates the surface soil and moves laterally through the upper soil horizons toward the streams. The *groundwater runoff* is that part of the runoff due to deep percolation of the infiltrated water, which has become groundwater, and has been discharged into the stream.

Total runoff in stream channels is generally classified as direct runoff and base flow. The direct runoff is that part of runoff which enters the stream promptly after the rainfall or snow melting. It is equal to the sum of surface runoff and the prompt subsurface runoff, plus channel precipitation. The base flow or base runoff is composed of groundwater runoff and delayed subsurface runoff.

5.1 Affecting factors

The runoff process in a drainage basin is influenced by climatic and physiographic factors. Climatic factors include the effects of precipitation, evaporation, interception, and transpiration. Physiographic factors are basin characteristics and channel characteristics. Basin characteristics include size, shape, and slope of the drainage area, permeability and capacity of groundwater formations, presence of lakes and swamps, and land use. Channel characteristics are related mostly to hydraulic properties of the streams.

The basic affecting factors cause most large drainage basins to behave differently from most small drainage areas. Namely, two basins of nearly the same size may behave entirely differently in runoff phenomena. One drainage basin may show prominent channel storage effects, like most large basins, while the other may manifest strong influence of the land use, like most small basins. Also, small basins are very sensitive both to high-intensity rainfalls of short duration and to land use.

In hydrological analysis and design, it is often necessary to develop relations between the precipitation and runoff by using the factors affecting runoff. In this section only the relation between rainfall and runoff is discussed. The reader is directed to the references at the end of this chapter for detailed information on runoff from snowmelt.

5.2 Rational method

The relation between rainfall and peak runoff is presented by many empirical and semi empirical formulas. The *rational formula* can be taken as representative one of such formulas.

$$Q=CiA \quad (5.1)$$

where Q is peak discharge in cubic feet per second (cfs), C is runoff coefficient depending on the characteristics of the drainage area, i is uniform rate of rainfall intensity in inches per hour (in/hr) for a duration equal to the time of concentration, and A is drainage area in acres. The conversion factor in using this equation from English to SI units is 0.278. The runoff coefficient varies from 0.05 for flat sandy surfaces, to 0.95 for asphalted and urbanized areas. Some hydrologists suggest increase of this coefficient

by 10-25% in cases of rainfalls with return periods of 25, 50 and 100 years. The runoff coefficient is the same for all storms on a given watershed. The maximum rate of flow computed with this formula is produced by the rainfall which is maintained for a time equal to the time of concentration of flow at the point of consideration. The *time of concentration* is the time required for the surface runoff from the remotest part of the drainage basin to reach the point being considered. The time of concentration is generally greater than the lag time of the peak flow. The most frequently used formula for the time of concentration in hours is given by *Kirpich* (1940):

$$t_c = 0.00013 \frac{L^{0.77}}{S^{0.385}} \quad (5.2)$$

where L is the length of the basin area in miles, and S is average slope of the basin in dimensionless ratio.

5.3 Hydrograph method

A graph showing stage, discharge, depth or other properties of flow with respect to time is known as a *hydrograph*. When the stage is plotted against time, the graph is *stage hydrograph*, which is usually obtained as recording chart from a recording gauge station (limnigraph). When the discharge is shown against time, the graph is *discharge hydrograph*, or simply a *hydrograph*. The shape of the hydrographs depend of many variables (form of the basin area, geology, climate, rainfall intensity, rainfall duration, land use, water use), but still they tend to adopt a characteristic shape. As a rule, the runoff collects relatively rapidly after the rainfall start and the hydrograph has steep rise (rising limb). This is followed by a longer recession curve after the rainfall cases. The recession curve is more or less independent of the time variations in rainfall and infiltration. It may be slightly dependent on areal rainfall distribution and heavily dependent on ground conditions. The general shape of the runoff hydrograph is presented in Figure 5.1.

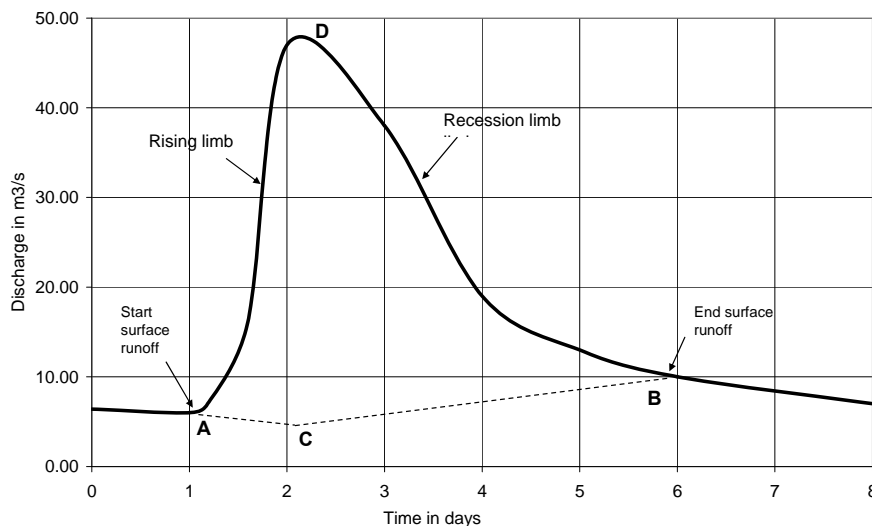


Figure 5.1 Typical river hydrograph

The peak of the hydrograph (point D) represents the highest concentration of the runoff from a drainage basin. It occurs usually at a certain time after the rain has ended, and this time depends on the areal distribution of the rainfall. The multiple peaks of a hydrograph may occur as the result of multiple storms being developed close to each other. The dashed line ACB in Figure 5.1 is called *base flow*, and it represents the groundwater discharge. Since the base flow is usually a relatively small portion of the total discharge during a period of storm runoff, the precise location of the line ACB is not a matter of great concern. The direct runoff is the area ACBD. The location of point B (surface runoff end) may be obtained by estimation of the time T_r between the peak and the end of direct runoff, which is given approximately by the expression:

$$T_r = 0.827A^{0.2} \quad (5.3)$$

where A is basin area in square miles or square kilometers respectively. Near the recession end, the observed hydrograph is modified to follow the normal recession curve, which is essentially a fitted smooth exponential curve.

Numerous methods of hydrograph analysis have been proposed and developed: *Sherman's* unit hydrograph, *Zoch's* theoretical analysis, *Snyder's* synthetic unit hydrograph, and others. The unit hydrograph method developed by *Sherman* and extended by many others will be discussed in the remaining portion of this section.

Hydrographs may be plotted with measured discharges, such as hydrograph on maximum discharges for a certain period, Figure 5.2, hydrograph on minimum discharges, or hydrograph on average discharges.

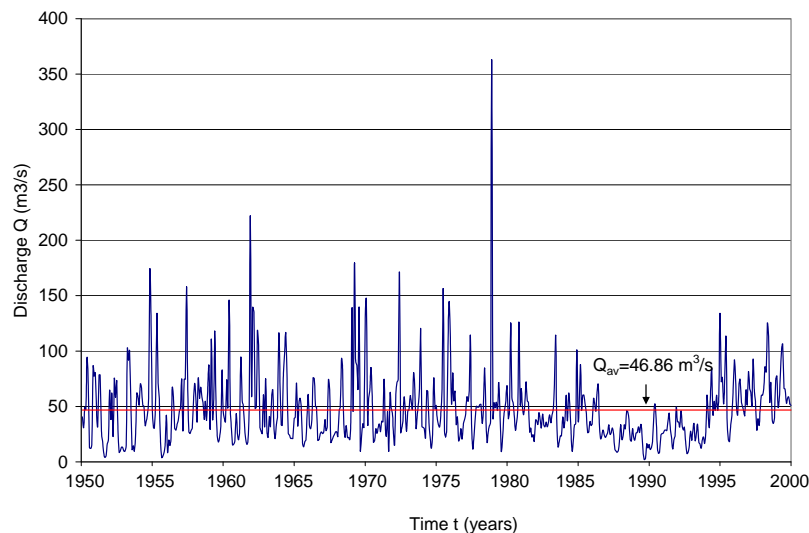


Figure 5.2 Hydrograph on maximum measured discharges (Upper Vardar River-Radusha)

5.4 Unit hydrograph

The unit hydrograph is the hydrograph of 1 cm of runoff storm of specified duration and distributed uniformly over the drainage area. The basic assumptions in unit hydrograph theory are:

1. All storms of the same duration have identical duration of surface runoff on a given drainage basin. Thus, the period of runoff is dependent on basin characteristics and the duration of the rainfall, but independent of the actual volume of runoff from the storm.
2. Two storms of identical duration and uniformly distributed over the basin will produce similar direct runoff hydrographs. This is the principle of linearity.
3. The direct runoff hydrograph from I given storm is independent of antecedent storms and the possibly concurrent runoff from them. This is the principle of superposition.

The procedure in estimating unit hydrographs involves the following basic steps:

1. Establish the design storm rainfall hyetograph.
2. Convert the design storm rainfall hyetograph into effective rainfall (runoff) hyetograph by subtraction of rainfall abstractions.
3. Estimate the unit hydrograph for the drainage basin.
4. Merge the effective rainfall hyetograph (runoff) and the unit hydrograph using discrete convolution to produce a direct runoff hydrograph, which represents the surface runoff hydrograph for the complex storm pattern represented by the rainfall hyetograph.
5. Add base flow to the direct runoff hydrograph to obtain the total hydrograph representing both surface runoff and base flow.

The first step in calculating the runoff hydrograph from a given rainfall hyetograph is to calculate the effective rainfall hyetograph. This can be done by subtraction of the interception from the beginning of a gross rainfall hyetograph, then subtraction of the infiltration from what remains after interception has been accounted for, and subtraction of the depression storage from what remains after both interception and infiltration have been accounted for. This approach is physically based and is not limited to use with any particular storm duration or rainfall hyetograph shape.

5.5 Synthetic hydrograph

Most commonly, rainfall and runoff records for a drainage basin do not exist, and one must resort to synthesis of unit hydrograph based on information that can be gathered about the basin. Extensive literature exists on various ways of calculating the synthetic hydrograph and the most commonly used are those proposed by *Snyder* (1938) and NRCS (SCS, 1969). The parameters that have to be calculated to design the direct runoff hydrograph are:

T_p -rising time from $Q=0$ to $Q=Q_{max}$

T_r -recession time, $T_r=kT_p$ ($k \geq 1$)

T_B -time of direct runoff (base of the hydrograph)

T_k -time of the effective rainfall duration

T_p -time from the center of gravity of the rainfall hyetograph to the peak of the hydrograph

T_c -time of concentration, or time from the center of gravity of the rainfall hyetograph to the center of gravity of the hydrograph ($T_c=T_0$)

The rising time is calculated with the rainfall duration time T_k and concentration time T_c by the following expression:

$$T_p = \left(\frac{T_k}{2} + T_0 \right) \frac{3}{k+2} \quad (5.4)$$

where k is coefficient dependent of the drainage basin characteristics $k=f(A)$, ($k \geq 1$). The time of concentration may be calculated by empirical formulas:

Kennedy&Watt:
$$T_c = 1.864 \cdot A^{0.39} S_s^{-0.31} \quad (5.5)$$

where A is drainage area in (km^2), S_s is average slope of the drainage basin in (‰) and the time is in (hours).

Kirpich:
$$T_c = 0.00025 \left(\frac{L}{\sqrt{S}} \right)^{0.8} \quad (5.6)$$

where L is the length of the drainage basin in (m), and S is average drainage basin slope. The volume of the direct runoff hydrograph may be computed with the drainage basin parameters ($W=P_e A$) or with the geometry of direct runoff hydrograph ($W=Q_{max} T_B/2$), from where the maximum discharge of direct runoff (peak of the hydrograph) may be computed:

$$Q_{max} = \frac{2P_e A}{T_B} \quad (5.7)$$

where P_e is effective rainfall in (mm), A is drainage basin area in (km^2), and T_B is time of direct runoff in (h).

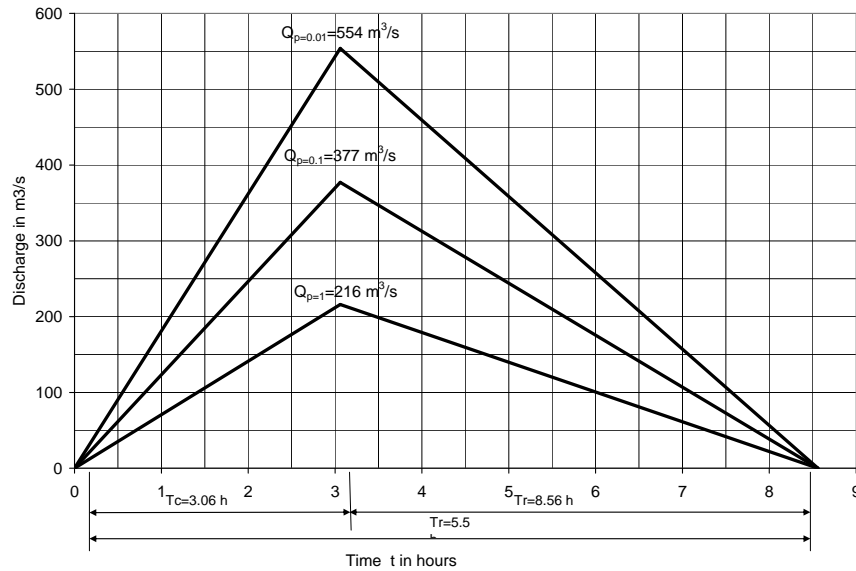


Figure 5.2 Synthetic hydrograph for different probability (River Lipkovska-LIPKOVO Dam)

5.6 Return period

The primary objective of the frequency analysis of hydrological data is to determine the recurrence interval of the hydrologic event of a given magnitude x . The average interval of time within which the magnitude x of the event will equaled or exceeded once is known as *recurrence interval*, *return period*, or simply the *frequency*, to be designated by T . If p is probability that the event will be equaled or exceeded in a particular year, the return period may be expressed:

$$T = \frac{1}{p} \quad (5.8)$$

One should keep in mind that a return period of a certain event e.g. 10 years does not imply that the event occurs at 10 year interval. It means that the probability that certain value is exceeded in a certain year is 10%. Consequently, the probability that the event does not occur (the value is not exceeded) is 90%. In general, the probability that an event with return period T actually occurs (once or more often) during n years period is:

$$p = 1 - \left(1 - \frac{1}{T}\right)^n \quad (5.9)$$

A frequency analysis to derive intensity-duration-frequency curves requires a length of record of at least 20-30 years to yield reliable results. For the territory of Macedonia *Skoklevski & Todorovski* (1993) have obtained these curves by using rainfall data from main rain gauge stations for the period 1956-1988. When designing structures different return period may be used and this is regulated by national legislation. Recommended design criteria in Macedonia are presented in Table 5.1.

Construction/Structure	Return period
Large dams with urban areas downstream	10000
Large dams with non-urbanized areas downstream	1000-10000
Small dams	100-1000
Culverts	10-25
Drainage systems	25-50
Bridges	50-100
Storm sewer systems	2-10

Table 5.1 Design criteria to return period

Example 5-1:

In drainage basin with rectangular shape and area $A=3 \times 10=30 \text{ km}^2$ is registered hydrograph due to surface runoff with volume $W=354000 \text{ m}^3$ due to rainfalls with duration and intensity as it is shown in the table below.

t (from-to) (h)	0-1	1-2	2-3	3-4
i (mm/h)	10	13	9	7

Determine the following: a) hyetograph of the effective rainfalls if the runoff coefficient is constant; b) hydrograph of surface runoff if the velocity flow in river is 2.5 km/h and the velocity of surface runoff is 1.5 km/h ; c) plot graphically the hydrograph and define its parameters (concentration time, recession time, volume of the hydrograph, peak of the hydrograph).

6. Flow routing models

Upon completion of this lesson, the students will be able to calculate the discharge downstream along a watercourse, as well as to estimate the discharge in various stages of flood-wave propagation.

The process of quantitatively following the hydrograph downstream and through reservoirs is called *flow routing*. Frequently, flood conditions are great concern and consequently the term *flood routing* is also used. The process of flood routing is extremely important because it provides prediction of magnitude and arrival time of a flood peak discharge. Routing also plays an important role in optimizing the design and operation of reservoir systems. The passing of hydrograph is unsteady flow when flow parameters (velocity, depth) change in time. Consequently, the most exact approach in solution the flow routing is to combine the unsteady flow equations from fluid mechanics, modified to describe the open channel conditions, with the continuity equation. The problem is rather complex, particularly when applied to natural river courses. Another, less-exact approach is the use of continuity equation alone. This procedure requires keeping track of the inflow, outflow, and the change in storage in a water body. Thus, the term *storage routing* is often used to describe this technique. Only the storage routing will be considered in this chapter. However, the process will be applied to the passage of flood waves through both reservoirs and river channels.

If the inflow is designate by I and the outflow by O , then the continuity equation may be expressed by:

$$(I - O)\Delta t = \Delta W \quad (6.1)$$

where Δt is time period, and Δs is the resulting change in storage. This equation may be expressed in finite difference form as:

$$\frac{(I_1 + I_2)}{2} \Delta t - \frac{(O_1 + O_2)}{2} \Delta t = W_2 - W_1 \quad (6.2)$$

where the subscripts 1 and 2 identify the various quantities I , O and W at the start and end of time period Δt is time period. This equation is most accurate for relatively short time intervals. The terms I and O have units of discharge (m^3/s), while $W/\Delta t$ will be consistent if the storage is in (m^3) and the time in seconds (s).

Routing of a hydrograph down river is complicated by the following additional factors: a) the storage in a reach of a river is not merely a function of the outflow as it is for reservoir routing; b) the storage can not be determined from a contour map as it can be done with reservoirs. So, with these complications there is a need of second relationship for the storage equation. If x is introduced as a weighting factor that indicates the relative importance of the inflow on the storage, then the channel storage equation is written:

$$W = \frac{b}{a^{m/n}} [xI + (1-x)O]^{m/n} \quad (6.3)$$

where a , b , m and n are constants for a given river. The most common river routing method is the *Muskingum* method. This approach assumes that $m/n=1$ and $b/a=K$, and the previous equation simplifies to:

$$W = K[xI + (1-x)O] \quad (6.4)$$

The storage W (obtained by adding the respective values of ΔW) can be plotted versus the weighted discharge $[xI+(1-x)O]$ for selected value of x . When $x=0$ the storage is dependent solely on the outflow (the reservoir case), while $x=0.5$ the inflow and outflow are equal. The best x is the value that results in the most nearly linear plot, and it is in the range $x=0.2$ to 0.3 . Once the x value has been identified, K is determined from the slope of the best fit line, Figure 6.1. If the last equation will be substituted into the finite difference continuity equation to eliminate the storage terms W_1 and W_2 , and solving for O_2 , the river routing equation becomes:

$$O_2 = c_1 I_1 + c_2 I_2 + c_3 O_1 \quad (6.5)$$

where the coefficients are given by:

$$c_1 = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t} \quad c_2 = -\frac{Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t} \quad c_3 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t} \quad (6.6-6.8)$$

As a check these coefficients must satisfy $c_1+c_2+c_3=1$. Once the hydrograph is routed through a single reach, the outflow hydrograph that has just been determined will serve as the inflow hydrograph for the next reach. In this way the routing process can be carried downstream similarly.

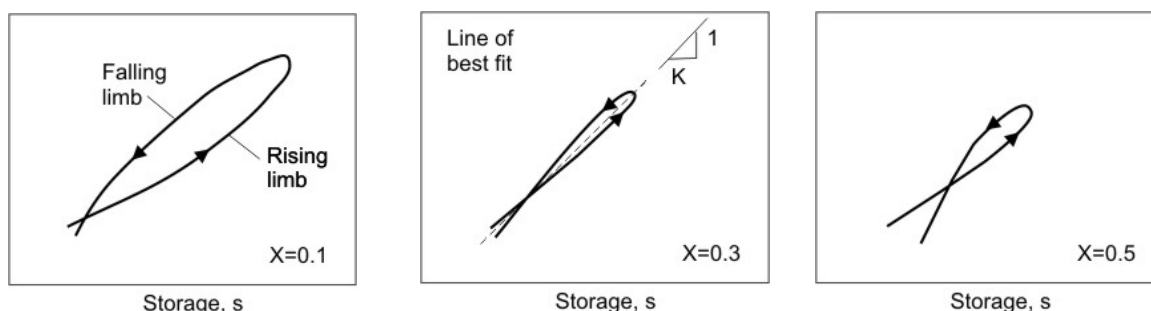


Figure 6.1 Determination of Muskingum constants

Example 6-1:

Route the inflow hydrograph through a reach of a river where it has been determined that $x=0.3$ and $K=0.9$ days if the initial inflow is $142 \text{ m}^3/\text{s}$.

T (days)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
Q (m^3/s)	142	187	396	850	1014	932	765	603	484	391	331	286	252

7. Flow duration curve

Upon completion of this lesson, the students will be able to construct a flow duration curve from a set of hydrological data.

In the study of surface waters the discharges and related questions on frequency and duration of normal flows and extreme flows are of primary importance. The flow duration curve is relationship between any given discharge and the percentage of time that the discharge is exceeded. The flow duration curve only applies for the period for which it was derived. If this is a long period, say over 20 years, the flow duration curve may be regarded as a probability curve, which may be used to estimate the percentage of

time that a specified discharge will be equalled or exceeded in the future. The range of discharges are divided into class intervals (c.i), then the number of periods (days, months, years) in which the flow belongs to the respective class intervals are counted. After that the cumulative totals of the number of periods and cumulative percentages are presented as it is shown in Table 7.1.

Class interval/ lower bound	Total in class interval	Number greater than bottom of class interval	Percentage greater than bottom of class interval
1500	2	2	10
1400	1	3	15
1300	0	3	15
1200	1	4	20
1100	1	5	25
1000	1	6	30
900	2	8	40
800	2	10	50
700	2	12	60
600	2	14	70
500	2	16	80
400	4	20	100

Table 7.1 Derivation of flow duration curve

A plot of discharge against the percentage of time that the discharge is exceeded shows the typical shape of flow duration curve. In Figure 7.1 is presented the flow duration curve for the example in Table 7.1.

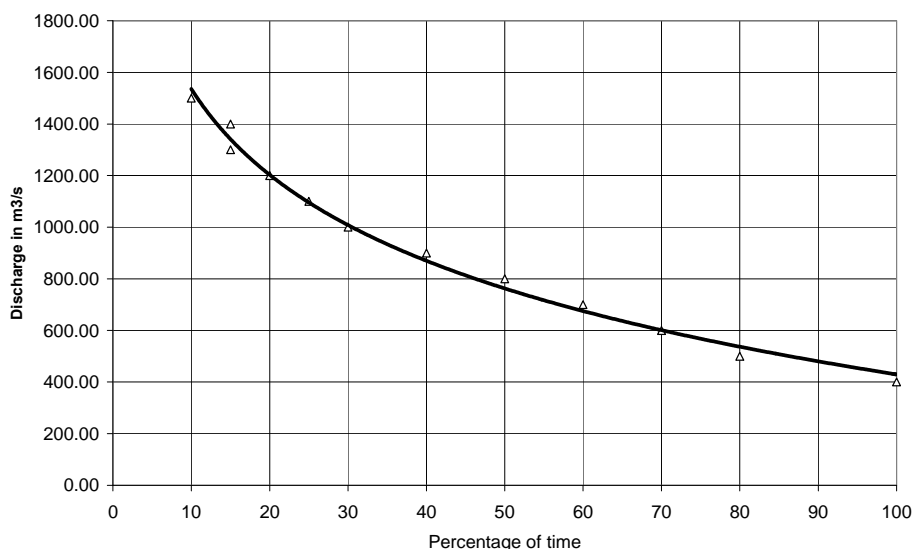


Figure 7.1 Flow duration curve

8. Flow rating curve

Upon completion of this lesson, the students will be able to construct a flow rating curve for a particular cross-section of a watercourse or canal.

The relationship between the water level and river discharge is called a *rating curve*, Figure 8.1. This is usually obtained in a stretch of the river where the riverbed is stable and the flow is uniform. Such conditions may be created artificially by constructing a control structure (threshold) across the river. The rating curve established at the gauging station has to be updated regularly, because scour and

sedimentation of the river bed and river banks may change the stage discharge relation, particularly after a flood. The rating curve equation is often represented by:

$$Q = a(H - H_o)^b \quad (8.1)$$

where Q is discharge in (m^3/s), H is water level in (m), H_o is water level at $Q=0$, and a and b are constants which are found by a least square fit using the measured data. The coefficient b depends of the geometry of the river cross section, and according to Water Resources Board (1965) it is 1.59 in a rectangular channel, 1.69 in a trapezoidal channel, 2.16 in a parabolic channel, and 2.67 in a triangular channel. This equation is compatible with the *Manning's* formula:

$$Q = \frac{A}{n} R^{2/3} S^{1/2} \quad (8.2)$$

where A is the cross-sectional area in (m^2), R is hydraulic radius in (m), and S is hydraulic gradient in (m/m). In steady uniform flow the hydraulic gradient is equal to the riverbed slope, $S=S_o$. The hydraulic radius R is obtained with cross section area and wetted perimeter ($R=A/P$).

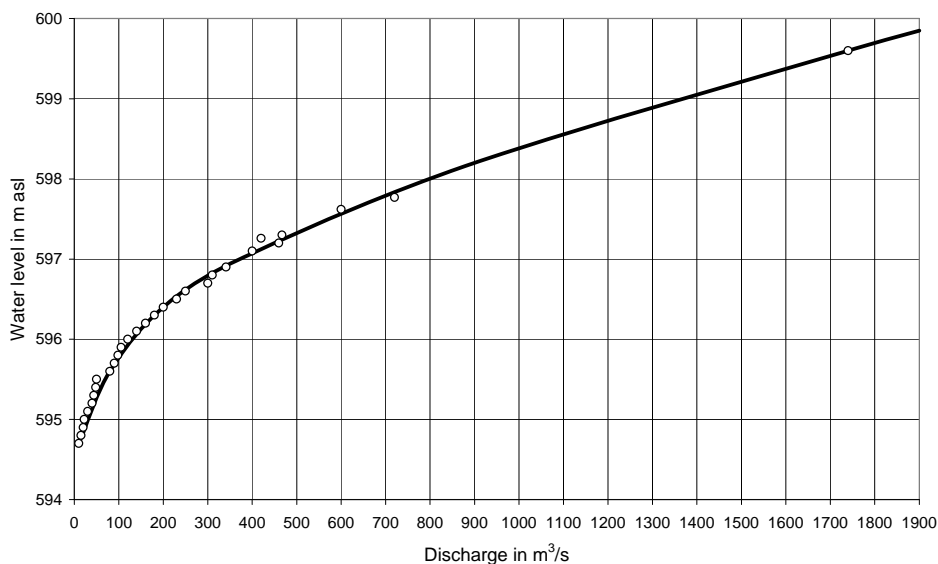


Figure 8.1 Flow rating curve

9. Floods and droughts

Upon completion of this lesson, the students will be acquainted with basics on hydrological floods and droughts.

9.1 Floods

Flood is relatively high flow that overtops the natural or constructed river banks. When banks are overtopped, water spreads over the flood plain and in general comes into conflict with man. Since the flood plains are often taken with man activities, the floods should be controlled to reduce the damage. Floods vary from month to month and year to year. The first step in forecasting the floods is to measure them. If the measurement is done for a long period analytical studies can be applied in flood analysis. Flood discharges may also be determined by analysis of hydrological data or by empirical formulas. These methods may consider precipitation, land use, watershed characteristics, and other parameters. Apart from normal flow frequency analysis, hydrologists are interested in the occurrence of flood events. For this purpose flow frequency curves may be derived which yield the probability that a certain annual maximum discharge is exceeded. Statistical methods are used and depending of the phenomenon,

different probability distributions are recommended, such as *Gumbel type I*, *Log-Gumbel*, *Pearson*, *Log-Pearson*, and others.

In a given period of n years, the probability of a given number k of events of a given return interval T is given by the binomial distribution as follows. In the limit of long periods (as n grows large), this converges to the Poisson distribution. Take

$$\frac{1}{T} = \frac{m}{n+1} \quad (9.1)$$

where T is return interval, m is ranking and n is number of occurrences

If the probability of a flood event occurring is p , then the probability of the event not occurring is $q = (1 - p)$. The binomial distribution can be used to find the probability of occurrence of an event r times in a period of n years.

$$P_T = \bar{P} + \sigma K_T \quad (9.2)$$

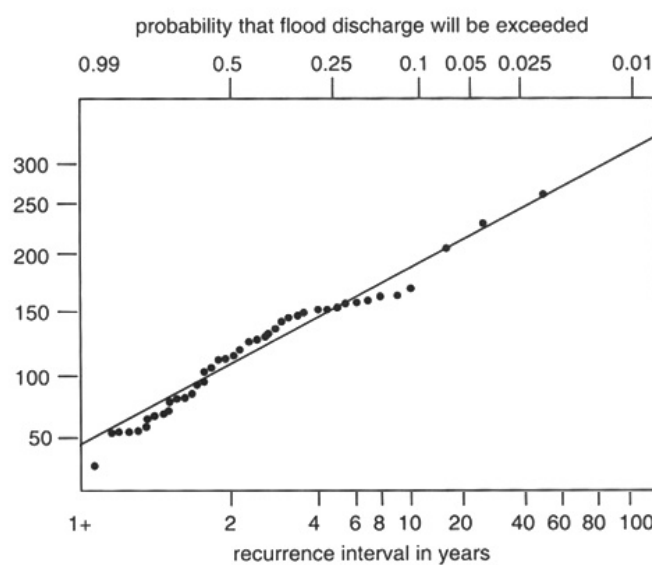


Figure 9.1 Example of plot of flood frequencies for a river

In frequency analysis of floods *Hershfield* (1961) introduced the so called **Maximum Probable Flood (MPF)**. This method is statistical empirical method based on the use the series of maximum daily precipitation (MPP_{24}). The following equation is used:

$$P_T = \bar{P} + \sigma K_T \quad (9.3)$$

where P_T is maximum annual precipitation for return period T , \bar{P} is arithmetic mean of the maximum annual precipitation data, σ is standard deviation of maximum annual precipitation data series, and K_T is frequency factor depending on return period T and applied probability distribution function. From mathematical prospective $K_T \rightarrow \infty$ when T is increasing. But in practice *Hershfield* obtained the maximum value of this factor between 3 and 15 by analysing over 2600 rain gauge stations.

9.2 Risk analysis

Return period is also useful for risk analysis (such as hydrologic risk of failure of a structure). When dealing with a structure design expectations the return period is useful in calculating the risk of the structure with respect to a given storm return period when given the design life expectation.

This is the likelihood of *at least one* event that exceeds design limits during the expected life of the structure: it is the complement of the likelihood that *no* events exceed design limits.

The equation for assessing this risk can be expressed as:

$$\bar{R} = 1 - \left(1 - \frac{1}{T}\right)^n = 1 - (1 - P(X \geq x_r))^n \quad (9.4)$$

where $\frac{1}{T} = P(X \geq x_r)$ is the expression for the probability of the occurrence for the hydrologic event in question; n is the expected life of the structure.

9.3 Droughts

Another extreme event in rivers is the occurrence of droughts. There are many definitions for drought. Some hydrologists define this event as low discharge, usually less than the average annual discharge, which lasts long period. Some others will recognize the drought as long period without precipitation. Similar as it was discussed for flood frequency analysis, for minimum flows may be developed the flow frequency curves which will give the probability of occurrence of annual minimum less than a given discharge. The statistical methods may be applied, and for droughts analysis the most recommended probability distribution is the *Log-Gumbel type III*.

10. Statistics in hydrology

Upon completion of this lesson, the students will be acquainted with basics on statistics used in the analyses of hydrological phenomena. Ultimately they will be able to statistically analyse hydrological data sets.

Hydrological data may be classified into experimental data and historical data. The experimental data are measured and can be obtained repeatedly by experiments. The historical data are collected from natural phenomena that can be observed only once and then will not occur again. Most hydrological data are historical data. Statistics deals with the computation of collected data, and probability deals with the measure of chance based on the collected data. The most important statistical parameters in hydrological analysis are defined by mean, median, mode, standard deviation, variance, coefficient of variation, and coefficient of asymmetry. There are three kinds of mean: arithmetic, geometric, and harmonic. The arithmetic mean, or simply *the mean*, is obtained by:

$$\bar{x} = \frac{\sum x}{N} \quad (10.1)$$

where x is the variate and N is the total number of observations. The *geometric mean* is the N th root of the product of N terms:

$$\bar{x}_g = (x_1 \cdot x_2 \cdot x_3 \cdots x_N)^{1/N} \quad (10.2)$$

The *harmonic mean* is obtained by the following reciprocal value:

$$\bar{x}_h = \frac{N}{\sum (1/x)} \quad (10.3)$$

The *median* is the middle value of the variate which divides the frequencies in distribution into two equal portions. *The mode* is the variate which is occurs most frequently. The *standard deviation* is a measure of variability and is computed by:

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}} \quad (10.4)$$

The square root of the standard deviation is called *variance*. The standard deviation divided by the mean is called the *coefficient of variation*, or:

$$C_v = \frac{\sigma}{\bar{x}} = \sqrt{\frac{\sum (K_i - 1)^2}{N}} \quad (10.5)$$

where K_i is the population mean ($K_i = x_i / \bar{x}$). The statistical parameter which measures the symmetry of distribution is called the *coefficient of asymmetry* and is computed by:

$$C_s = \frac{\sum (K_i - 1)^3}{N \cdot C_v^3} \quad (10.6)$$

For a shorter historical data series this coefficient may be computed by $C_s = 2C_v$. For symmetrical distribution $C_s = 0$, and when $C_s > 0.5$ the asymmetry is great. In practice the asymmetry is often obtained by the Pearson distribution:

$$C_s = \frac{2C_v}{1 - K_{min}} \quad (10.7)$$

10.1 Regression and correlation

The oldest statistical tools in hydrology are the regression and correlation analysis. It is used to fill missing data, to extend short records, to establish relationship between two or more hydrological variables, and to investigate the dependence between two successive values of hydrological data series. When only two variables are related the analysis is simple regression or correlation. When three or more variables are involved the analysis is multiple regression or correlation.

The most often used simple regression and correlation is linear one which a special case of curvilinear regression and correlation. The straight-line regression for variable y versus variable x is defined by a straight line which gives the best estimate of y for a given value of x . The straight regression line is generally fitted analytically by the method of least squares of the departures from the line. The regression equations are:

$$\begin{aligned} y - \bar{y} &= R_{y/x} (x - \bar{x}) \\ x - \bar{x} &= R_{x/y} (y - \bar{y}) \end{aligned} \quad (10.8)$$

where \bar{x} and \bar{y} are the arithmetic means of the series, $R_{x/y}$ and $R_{y/x}$ are the regression coefficients. These two lines are crossing at the point with coordinates \bar{x} and \bar{y} . The regression coefficients are obtained by the expressions:

$$\begin{aligned} R_{y/x} &= r \frac{\sigma_y}{\sigma_x} \\ R_{x/y} &= r \frac{\sigma_x}{\sigma_y} \end{aligned} \quad (10.9)$$

where r is a correlation coefficient, and σ_x and σ_y are the standard deviations of the series x and y , respectively. The correlation coefficient is statistical parameters which measures the degree of association of linearly dependent variables and is computed from the expression:

$$r = \frac{\sum_{i=1}^{i=n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{i=n} (x_i - \bar{x})^2 \sum_{i=1}^{i=n} (y_i - \bar{y})^2}} = \frac{\sum_{i=1}^{i=n} (\Delta x \Delta y)}{\sqrt{\sum_{i=1}^{i=n} (\Delta x)^2 \sum_{i=1}^{i=n} (\Delta y)^2}} \quad (10.10)$$

The sign of r depends on the sum of the cross products $\Delta x \Delta y$ and it varies from +1 to -1. If the correlation coefficient is zero, the variables x and y are linearly independent. A positive value of r means that y increases with increase of x . A negative value of r means that y decreases with increase of x . The standard deviations of the regression lines y versus x and x versus y are defined:

$$S_y = \sigma_y \sqrt{1 - r^2} \quad (10.11)$$

$$S_x = \sigma_x \sqrt{1 - r^2}$$

The greater S_y and S_x are, the wider is the spread of the points around the regression line and the less accurate are the values determined from the regression lines. The standard deviation of the correlation coefficient is obtained:

$$S_r = \pm \frac{1 - r^2}{\sqrt{N}} \quad (10.11)$$

10.2 Curve fitting

The curve fitting in hydrology may be graphical or analytical. The graphical method is based on tracing a curve by eyes through the mean of the spread of the plotted points $y=f(x)$ or $y=f(x, a, b, c, \dots)$. The currently approved analytical method of fitting curves is to minimize the sum of squares of differences $\Delta y = y - y_i$, where for a given x_i , the value y is determined from the fitted curve, and y_i is from the observed data. This is known as the *least-square method*. All partial derivatives of the sum of squares of departures with respect to the parameters a, b, c, \dots should be zero:

$$\frac{\partial \sum_{i=1}^{i=n} (y - y_i)^2}{\partial a} = 0 \quad \frac{\partial \sum_{i=1}^{i=n} (y - y_i)^2}{\partial b} = 0 \quad \frac{\partial \sum_{i=1}^{i=n} (y - y_i)^2}{\partial c} = 0 \quad (10.12)$$

For example, the fitting of a quadratic parabola between the measured discharge Q and water level H is given by the equation:

$$Q = f(H) = cH^2 + bH + a \quad (10.13)$$

By obtaining the derivatives of the sum $\sum (Q - Q_i)^2$ with respect to unknown parameters a, b and c , the following three equations may be written:

$$\sum Q_i = c \sum H^2 + b \sum H + aN \quad (10.14)$$

$$\sum Q_i H^2 = c \sum H^4 + b \sum H^3 + a \sum H^2 \quad (10.15)$$

$$\sum Q_i H = c \sum H^3 + b \sum H^2 + a \sum H \quad (10.16)$$

with the sums taken from $i=1$ to N . The solution of these three equations gives a, b and c . Other analytical functions that are used in hydrology are:

$$y = ax + b \quad (10.17)$$

$$z = ax + by + c \quad (10.18)$$

$$y = ax^b \quad (10.19)$$

The last function is exponential, and its logarithmic transformation is frequently used in hydrology:

$$\log y = \log a + b \log x \quad (10.20)$$

Problem 10-1:

For the measured discharges Q and water stages H at hydrological station on a river, determine analytically and graphically the rating curve by the method of least-square.

H (cm)	170	182	190	213	230	241	264	279	293	300	321	345	360
Q (m ³ /s)	71	80	98	122	159	162	203	230	254	260	287	342	350

10.3 Probability distributions

Apart from normal flow frequency, the occurrence of extreme events is also interest of hydrologists. For this purpose flow frequency curves may be derived which yield the probability that certain annual maximum discharge is exceeded. For minimum flows similar curves may be developed giving the probability of occurrence of an annual minimum less than a given discharge. Depending on the phenomenon, different probability distributions are recommended. For example, for droughts the *Log-Gumbel type III* distributions may be used, and for floods the *Gumbel type I*, *Log-Gumbel*, *Pearson* or *Log-Pearson type III* distributions.

Gumbel type I

Gumbel distribution (1941) has been used with success to describe the populations of many hydrological events. The fundamental theory can be stated as it follows. If $x_1, x_2, x_3, \dots, x_n$ are independent extreme values observed in n samples of equal size N (eg. years, months), and if x is an unlimited exponentially distributed variable, then as N and n approach infinity, the cumulative probability q that any of extremes will be less than a given value x_i is given by:

$$q = \exp(-\exp(-y)) \quad (10.21)$$

where q is the probability of non-exceedence, and y is the reduced variate. If the probability that x will be exceeded is defined as $p=1-q$, then the last equation yields:

$$y = -\ln(-\ln(1 - p)) = -\ln(-\ln(1 - \frac{1}{T})) \quad (10.22)$$

where T is the *return period* measured in sample sizes N (eg. years). According to *Gumbel* there is a linear relation between x and y :

$$y = ax^b \quad (10.23)$$

$$y = a(x - b) \quad (10.24)$$

where a is the dispersion factor and b is the mode. *Gumbel* showed that if the sample N goes to infinity:

$$b = \bar{x} - 0.45005\sigma \quad (10.25)$$

$$a = \frac{1.28255}{\sigma} \quad (10.26)$$

where \bar{x} is the mean of x , and σ is the standard deviation of the sample. If the sample is finite, which they always are, the coefficients a and b are adjusted according to the following equations:

$$b = \bar{x} - \sigma \frac{\bar{y}}{\sigma_y} \quad (10.27)$$

$$a = \frac{\sigma_y}{\sigma} \quad (10.28)$$

where \bar{y} is the mean of the reduced variate, and σ_y is the standard deviation of the reduced variate. These values are usually tabulated as a function of N . The linear relation between x and y thus is modified:

$$x = \bar{x} + \frac{(y - \bar{y})\sigma}{\sigma_y} \quad (10.29)$$

Data represented by this distribution on probability paper plot a straight line. The horizontal axis is probability of exceedence p , or non-exceedence q , or the recurrence interval T , on a non-linear scale. The vertical axis usually represents the analyzed values in certain scale.

Log-Gumbel type III

The *Log-Gumbel III* distribution is often quite adequate for analysis of extreme low flows (droughts). The probability of exceedence is computed by:

$$p = 1 - \frac{i - 0.25}{n + 0.5} \quad (10.30)$$

where i is the rank number in decreasing order and n is the number of observations. The reduced variate is computed by:

$$y = -\ln(-\ln(p)) \quad (10.31)$$

and the probability of non-exceedence is determined by:

$$q = 1 - \frac{i}{n + 1} \quad (10.32)$$

Consequently, the reduced variate is given by:

$$y = -\ln(-\ln(q)) \quad (10.33)$$

Pearson type III

Pearson (1930) derived a series of probability functions to fit virtually any distribution. Although these functions have only slight theoretical basis, they have been used widely in practice. The general equation to define the probability density of a Pearson distribution is:

$$p(x) = \int_{-\infty}^x \frac{(a+x)^{(a+x)/(b_0+b_1x+b_2x^2)} dx}{e} \quad (10.34)$$

where $a, b_0, b_1,$ and b_2 are constants. The criteria for determining the types of distribution are β_1, β_2 and $k,$ being defined as follows:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \quad (10.35)$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \quad (10.36)$$

$$k = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} \quad (10.37)$$

where $\mu_2, \mu_3,$ and μ_4 are the second, third, and fourth moments about the mean. The r th moment about the mean \bar{x} of the variates x_1, x_2, \dots, x_n is:

$$\mu_r = \frac{1}{N} \sum_{i=1}^{i=n} p_i (x_i - K_i)^r \quad (10.38)$$

where p_i is the frequency or probability of $x_i,$ and $N = \sum p_i$ with $i=1,2,\dots,n.$ For type III $k=\infty$ or $2\beta_2=3\beta_1+6.$ Its probability density is:

$$p(x) = p_0 \left(1 + \frac{x}{a}\right)^c e^{-cx/a} \quad (10.39)$$

where:

$$c = \frac{4}{\beta_1} - 1 \quad (10.40)$$

$$a = \frac{c \mu_3}{2 \mu_2} \quad (10.41)$$

$$p_0 = \frac{N}{a} \frac{c^{c+1}}{e^c \Gamma(c+1)} \quad (10.42)$$

In these expressions $\Gamma(c+1)$ is gamma function, and N is the total frequency. More about probability distributions and their tabulated parameters may be found in the references.